7. State Rolle's theorem and verify the Rolle's theorem for $f(x) = x^3 - x^2 - 6x + 2$ in [0, 3].

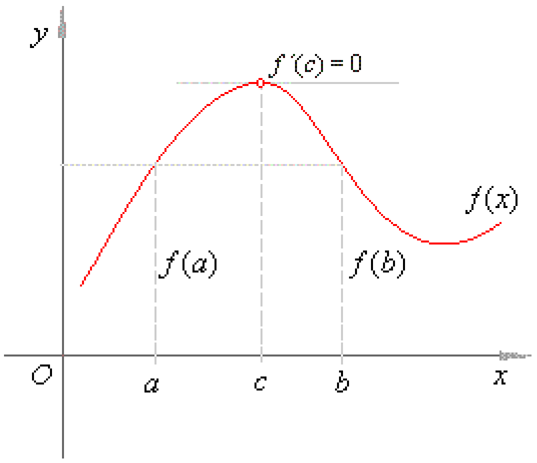
Solution:

here,

$$f(x) = x^3 - x^2 - 6x + 2 \quad \forall \quad x \in [0, 3].$$

Rolle's theorem:

A function f(x) is:



i. continuous on [a, b]
ii. differentiable on (a, b)
iii. f(a) = f(b)

Then, there exists at least a point c ∈ (a, b) such that,

$$\therefore \; f'(c) = 0.$$

Problem Part:

i. $f(x) = x^3 - x^2 - 6x + 2$ is an polynomial function and polynomial functions are continous on their domain. So, f(x) is continous $\forall \; x \in [0, 3]$.

ii. $f'(x) = 3x^2 - 2x - 6$ is defined $\forall \; x \in (0, 3)$. So, f(x) is differentiable $\forall \; x \in (0, 3)$.

iii. Thus, f(x) satisfies both conditions of Rolle's Theorem so, by Rolle's Theorem there exists at least a point c ∈ (0, 3) such that,

$$f'(c) = 0$$

or, $3c^2 - 2c - 6 = 0$

or, $c = \frac{1 \pm \sqrt{19}}{3}$

$\therefore c = \frac{1 + \sqrt{19}}{3} \in (0, 3)$

Hence, Rolle's Theorem is verified.